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Differentiation Technique - Standard Functions

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Differentiation

Subtopics: Vertical Tangents, Local or Relative Minima and Maxima, Concavity, Tangents To Curves, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 4

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   (c) Write an equation for the line tangent to the graph of h at $x = 4$.

   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Differentiation

Subtopics: Points Of Inflection, Local or Relative Minima and Maxima, Integration Technique – Standard Functions, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Medium / Question Number: 5

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k ?

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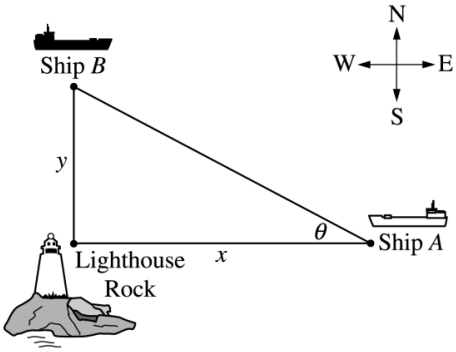
Question 3

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Rates of Change (Instantaneous), Related Rates, Implicit Differentiation, Modelling Situations, Differentiation Technique – Standard Functions, Differentiation Technique – Trigonometry, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Hard / Question Number: 6



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

   (a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

   (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

   (c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

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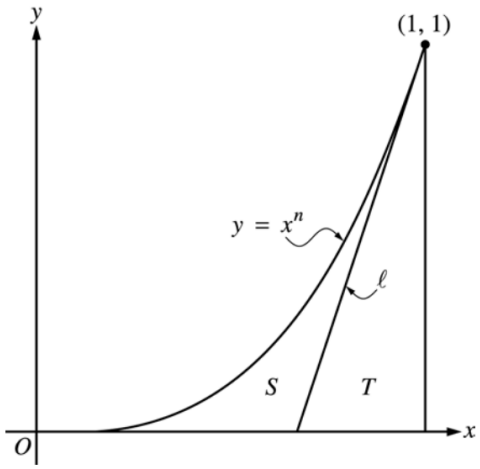
Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Standard Functions, Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004-Form-B / Difficulty: Hard / Question Number: 6



6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

   (a) Find $\int_0^1 x^n \, dx$ in terms of n.

   (b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

   (c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

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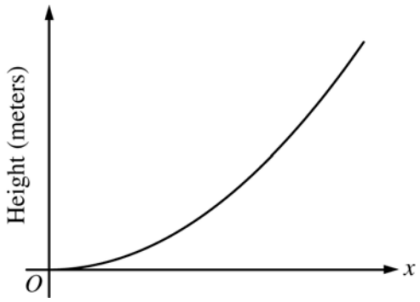
Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Increasing/Decreasing , Tangents To Curves, Differentiation Technique – Standard Functions

Paper: Part A-Calc / Series: 2006-Form-B / Difficulty: Somewhat Challenging / Question Number: 3



3. The figure above is the graph of a function of x, which models the height of a skateboard ramp. The function meets the following requirements.

   (i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.

   (ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.

   (iii) Between $x = 0$ and $x = 4$, the function is increasing.

   (a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.

   (b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.

   (c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.

   (d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.

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Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Differentiation Technique – Standard Functions, Local or Relative Minima and Maxima, Points Of Inflection

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Easy / Question Number: 6

6. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

   (c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

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Question 7

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Differentiation Technique – Standard Functions, Vertical Tangents

Paper: Part B-Non-Calc / Series: 2008-Form-B / Difficulty: Somewhat Challenging / Question Number: 6

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6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

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Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Concavity, Differentiation Technique – Standard Functions, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Medium / Question Number: 4

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

   (a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

   (b) Find all intervals on which the graph of f is concave down. Justify your answer.

   (c) Given that $f(1) = 2$, determine the function f.

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Question 9

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation, Differentiation, Integration

Subtopics: Differentiation Technique – Chain Rule, Differentiation Technique – Standard Functions, Tangents To Curves, Continuities and Discontinuities, Integration Technique – Substitution

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Medium / Question Number: 4

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4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

   (a) Find $f'(x)$.

   (b) Write an equation for the line tangent to the graph of f at $x = -3$.

   (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

   Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

   (d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

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